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Advanced Decision-Support Model for Systemic Lupus Erythematosus Diagnosis using Rough Topology Technique

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Abstract
This study presents a precise mathematical framework to support the diagnosis of systemic lupus erythematosus using topological rough set theory. Topological rough sets offer a robust framework for handling incomplete information within an information system using topological tools. Central to this theory are the notions of upper and lower approximations related to a defined universal set. The concepts of upper and lower approximation, associated with a comprehensive set of specific symptoms, are essential to this theory. We focus on applying them to real-life cases suspected of having lupus based on some of their symptoms. We propose several measures, such as accuracy, dependency quality, approximate membership, and attribute significance, to assess medical uncertainty. We apply these criteria to a clinical model consisting of clinical cases. This approach has demonstrated the ability to improve diagnosis, particularly in ambiguous cases, and to identify key symptomatic indicators, thus facilitating more accurate clinical decision-making.

Key words: Rough set, lower (upper) approximations, boundary region, rough topology, positive(negative) regions, accuracy of approximation, dependency degree, attributes importance, reduct and core.

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نموذج دعم القرار المتقدم لتشخيص الذئبة الحمامية الجهازية باستخدام تقنية الطوبولوجيا التقريبية

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الملخص

تقدم هذه الدراسة إطارًا رياضيًا دقيقًا لدعم تشخيص الذئبة الحمامية الجهازية باستخدام نظرية الطوبولوجيا التقريبية. توفر هذه النظرية إطارًا قويًا للتعامل مع المعلومات غير الكاملة ضمن نظام معلومات باستخدام أدوات طوبولوجية. ويُعدّ مفهوما التقريب العلوي والسفلي، المرتبطين بمجموعة شاملة محددة، أساسيين لهذه النظرية. ويركز البحث على تطبيق هذين المفهومين على حالات واقعية يُشتبه بإصابتها بالذئبة بناءً على بعض أعراضها. ونقدم عدة مقاييس مثل الدقة وجودة التبعية والعضوية التقريبية وأهمية السمة لتقييم عدم اليقين الطبي. ونطبق هذه المعايير على نموذج سريري يتكون من حالات سريرية. وقد أثبت هذا النهج قدرته على تحسين التشخيص، لا سيما في الحالات الغامضة، وتحديد المؤشرات العرضية الرئيسية مما يُسهّل اتخاذ قرارات سريرية أكثر دقة.

الكلمات المفتاحية: مجموعة تقريبية، تقريبات سفلية(علوية)، منطقة حدودية، طوبولوجيا تقريبية، مناطق موجبة (سالبة)، دقة التقريب، درجة التبعية، أهمية السمات، الاختزال، النواة.

1-Introduction

In the early 1980s, the Polish, mathematician Pawlak introduced rough set theory [1] as an innovative mathematical tool for dealing with uncertain and incomplete data. This theory is distinguished by its high capacity for analyzing and classifying ambiguous information by establishing precise boundaries through lower and upper approximations [2, 3]. With the evolution of this theory, topological concepts were integrated to form topological rough spaces [4, 5], where the lower approximation acts as an interior operator and the upper approximation acts as a closure operator within the topological space. The increasing and rapid development

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of topological analysis is enhancing its importance in practical applications, particularly in real-world fields, as clearly demonstrated in medical diagnosis. It possesses a high capacity for describing the qualitative characteristics of information spaces, transforming clinical ambiguity into analyzable mathematical structures. Topological closure provides precise mechanisms for identifying confirmed pathological cases and separating them from ambiguous cases located in the boundary region.

The significance of these mathematical tools is clearly demonstrated when applied to complex health crises, primarily systemic lupus erythematosus (SLE) [6]. This disease is defined as a chronic autoimmune disorder in which the body loses its ability to distinguish between its own tissues and foreign substances, leading to the production of antibodies that attack vital organs such as the skin, joints, kidneys, and heart [7, 8]. The clinical symptoms appear diversely and unpredictably, including butterfly-shaped facial rash, joint pain and swelling, photosensitivity, and hair loss, in addition to extreme fatigue and anemia [9, 10].

The greatest challenge in diagnosing SLE lies in its chameleon-like nature, as its symptoms significantly overlap with other autoimmune diseases, making clinical differentiation in the early stages highly complex despite international classification criteria like (ACR) and (SLICC) [11, 12, 13]. Therefore, there is an urgent need for methodologies capable of analyzing the boundary region between confirmed and suspected cases. Consequently, we employ this rough topological framework as the most suitable methodology to deconstruct diagnostic complexity and transform ambiguous clinical symptoms into precise mathematical data that support accurate medical decision-making.

This study aims to bridge the gap between theoretical rough topology and real-world medical diagnosis in practice. It presents a case study of ten patients whose medical records were reviewed at a medical center and who were suspected of having systemic lupus erythematosus (SLE). Some common symptoms (features) of the disease were selected based on internationally accepted clinical criteria for diagnosing SLE. Using tools of rough topology as (positive regions, accuracy of approximation, dependency degree, attributes importance, reduct and core), we explore the relationship

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between these features to identify the most important and indispensable symptoms that influence decision-making.

The paper is organized as follows: Section 2 presents the basics of Pawlak approximation space and information system. Section 3 provides the basic idea of rough topology. The medical information system for SLE is given in Sections 4. In Section 5, we apply measures of rough topology to the medical table for SLE, followed by a medical interpretation of the results obtained using all the tools. The research concludes with a conclusion in Section 6.

2- Pawlak Approximation Space and Information System:

Pawlak's approximation spaces are considered fundamental pillars of rough set theory, as they provide the essential basis for the approximation process. These spaces utilize equivalence relations to partition data into classes where elements are indiscernible based on available information. The theory's significance lies in its ability to eliminate ambiguity through an organized mathematical approach, making it highly effective for analyzing incomplete data.

Suppose that U be a non-empty finite set of objects, suppose that R an equivalence relation on$U$, the pair (U, R) is called Pawlak approximation space. We use U/R to denote the family of all equivalent class of R determined by x denoted by $([x]_R)$.

Definition 2.1[1] Let (U, R) be Pawlak approximation space and $X \subseteq U$, we define the lower and upper approximations of X respectively as

$\underline{R}(X) = \{x \in U : [x]_R \subseteq X\}$, $\overline{R}(X) = \{x \in U: [x]_R \cap X \neq \emptyset\}$

Remark 2.1 The upper approximation of X with respectively to R is the set of all objects, which can be possibly classified to$X$, while the lower approximation is all the objects are exactly in X.

Definition 2.2[1] Let (U, R) be Pawlak approximation space and $X \subseteq U$ is said to be

1. X is rough if and only if $\underline{R}(X) \neq \overline{R}(X)$.
2. X is definable (exact) if and only if $\underline{R}(X) = \overline{R}(X)$.

Definition 2.3[1]: For Pawlak approximation space (U, R) the boundary region of X with respect to R is defined by $B_R(X) =$

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$\overline{R}(X) - \underline{R}(X)$, which is the set of all objects, which can be classified in X neither in X^C where X^C is denoted the complement of X in U.

Remark 2.2: The set X is a rough with respect to R if and only if $B_R(X) \neq \emptyset$. Other that X is definable.

Definition 2.4[1]: Let (U, R) be Pawlak approximation space and $X \subseteq U$, the positive and negative regions of $X \subseteq U$ are given respectively, by

$$POS_R(X) = \underline{R}(X),\ NEG_R(X) = U - \overline{R}(X).$$

Proposition2.1 [**5**] Let (U, R) be Pawlak approximation space, X and Y are subsets of U then:

1. $\underline{R}(X) \subseteq \overline{R}(X)$.
2. $\overline{R}(\emptyset) = \underline{R}(\emptyset) = \emptyset,\ \overline{R}(U) = \underline{R}(U) = U$.
3. $\overline{R}(X \cup Y) = \overline{R}(X) \cup \overline{R}(Y)$.
4. $\underline{R}(X \cup Y) \supseteq \underline{R}(X) \cup \underline{R}(Y)$.
5. $\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$.
6. $\overline{R}(X \cap Y) \subseteq \overline{R}(X) \cap \overline{R}(Y)$.
7. $\overline{R}(X) \subseteq \overline{R}(Y),\ \underline{R}(X) \subseteq \underline{R}(Y)$ when ever $X \subseteq Y$.
8. $\underline{R}(X^c) = [\overline{R}(X)]^c$ and $\overline{R}(X^c) = [\underline{R}(X)]^c$.
9. $\underline{R}\,\underline{R}(X) = \overline{R}\underline{R}(X) = \underline{R}(X)$.
10. $\overline{R}\,\overline{R}(X) = \underline{R}\overline{R}(X) = \overline{R}(X)$.

Definition 2. 4[3]: Let (U, R) be Pawlak approximation space and $X \subseteq U$, the accuracy approximation $\alpha_R(X) = \frac{card\underline{R}(X)}{card\overline{R}(X)}$, where $card(X)$ denotes cardinality of the set X and $\overline{R}(X) \neq \emptyset$. This definition measures the "roughness" or "smoothness" of a set X within Pawlak approximation space.

Remark 2.3: For Pawlak approximation space (U, R) and $X \subseteq U$, value of the accuracy approximation is $(0 \leq \alpha_R(X) \leq 1)$. We distinguish three cases:

1- If $\alpha_R(X) = 1$, the lower approximation equals the upper approximation, meaning set X is definable (exact).

2- If $\alpha_R(X) < 1$, set X is rough, indicating uncertainty about its boundaries.

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3- If $\alpha_R(X) = 0$ the lower approximation is empty set. Which means there is not any element classified to X based on available knowledge.

Definition 2.5 [3]: Let P and Q be equivalence relations over U, we define
P-positive region of Q, denoted $POS_P(Q)$ as:
$POS_P(Q) = \cup_{Y \in U/Q} \underline{P}(Y)$
The P-positive region of Q is the set of all objects of the universe U which can be properly classified to classes of U/Q employing knowledge expressed by the classification U/P.
This definition describes how well a set of attributes P can classify another set of attributes Q (or determine the categories of Q based on P). In other words, these are objects with no ambiguity regarding their membership in $Q -$ categories when using knowledge derived from P.

Partial Dependency of Knowledge:
The derivation (dependency) can also be partial, which means that only part of knowledge Q is derivable from knowledge P. The partial derivability can be defined using the notion of the positive region of knowledge. We will now define the partial derivability formally.
Definition 2.6 [3]: Let $K = (U, R)$ be a knowledge base and $P, Q \subset R$. We say that knowledge Q depends on a degree $K (0 \le K \le 1)$ from knowledge P, symbolically $P \Rightarrow_K Q$, if and only if

$$K = \gamma\rho(Q) = \frac{\text{card} POS_P(Q)}{\text{card } U}$$

1. If $K = 1$, we will say that Q totally depends onP.
2. If $0 \le K \le 1$, we say that Q roughly (partially) depends onP.
3. If $K = 0$we say that Q is totally independent fromP. If $P \Rightarrow_1 Q$, we shall also write $P \Rightarrow Q$.
   It measures the extent to which a specific decision depends on a set of conditional attributes, and it helps in identifying the attributes with the least importance as well as determining the essential and non-essential attributes.

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Definition 2.7[3] : Information System
A knowledge representation system is a pair $S = (U, A)$, where U is a nonempty finite set, called the universe. A is a nonempty finite set of primitive attributes. The pair $S = (U, A)$ is called an information system.
If R is an equivalence relation over U, then by U/R we mean the family of all equivalence classes of R (or classification of U) referred to as categories or concepts of R, and $[x]_R$ denotes a category in R containing an element $x \in U$.
Definition 2.8 [3]: Let $S = (U, A)$ be an information system. The indiscernibility relation on S is defined as $IND(R) = \{(x, y) \in U^2 : \forall\, a \in R, a(x) = a(y)\}$.

Example2.1: We will provide a simple example of an information system in table(1), which illustrates the main symptoms of five patients with diabetes. We present some medical information about the patients.

Table (1): Medical information for some patients with diabetes

patient	Fasting glucose	BMI	HbA1c	Decision
p_1	High	Over weight	High	Diabetic
p_2	High	Obese	High	Diabetic
p_3	Medium	Normal	Medium	Pre-diabetic
p_4	High	Over weight	Medium	Pre-diabetic
p_5	Very High	Obese	High	Diabetic

The rows represent the patients, while the columns represent the attributes used in the description: fasting blood glucose level, glycated hemoglobin (HbA1c), and body mass index. These attributes were chosen because they are among the most important clinical indicators used in the diagnosis and monitoring of diabetes, and they reflect clear differences between patients in terms of disease severity and level of control.
This system demonstrates how to represent medical data in a structured way that allows for the comparison and differentiation of various cases based solely on available information.
For example, for patient p_3 the fasting glucose level is classified as medium, indicating a moderate elevation in blood glucose that does not yet reach the diabetic threshold. The BMI value is normal, which

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suggests that the patient has a healthy body weight and no obesity-related risk factors. In addition, the HbA1c level is medium, reflecting an average blood glucose level over the past few months that is higher than normal but not critically high. Based on the combination of these attributes, patient p_3 is classified as pre-diabetic. This indicates an intermediate health condition where the patient is at increased risk of developing diabetes in the future, but the disease is not fully established. From a rough set perspective, the patient's condition lies in an uncertain region between normal and diabetic states, which justifies the pre-diabetic decision.

Definition 2.9 [3] Let $S = (U, A)$ and $\mathbf{R}$ be a family of equivalence relations on U and $R \in \mathbf{R}$. We will say that R is dispensable in **R** if $IND(\mathbf{R}) = IND(\mathbf{R} - \{R\})$; otherwise, R is indispensable in **R**.
The family **R** is independent if each $R \in \mathbf{R}$ is indispensable in **R**; otherwise, **R** is dependent.

Definition 2.10 [**3**]: Let $S = (U, A)$ and **R** be a family of equivalence relations on U. $P, Q \subseteq \mathbf{R}$ such that $Q \subseteq P$, we say that Q is reduct of P if and only if $IND(Q) = IND(P)$. Therefore, a subset of attributes $Q \subseteq P$ it is the smallest set of attributes that produces the same indiscernibility relation as the full set P.

Objective to finding of reducts is:

To remove redundant data (attributes) while preserving the original classification power of the information system.

Definition 2.11 [**3**]: The intersection of all possible Reducts of P denoted $Core(P)$ is called the Core of P

Objective to finding of core is:

1. To identify the most critical attributes that cannot be removed without changing the classification structure.
2. Simplifying large datasets by keeping only essential attributes.
3. Extracting minimal and understandable rules from data.

Attribute Importance:

It is one of the most important and useful tools in (RST). It helps identify unnecessary attributes that can be removed without affecting diagnostic accuracy, as well as aiding in the identification of essential attributes. It also increases the accuracy of the decision

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model and is defined as the difference between the degree of reliability of a decision when a particular attribute is present and its degree of reliability when removed from the set of attributes.

$$SIG(RA, C, D) = \gamma_C(D) - \gamma_{C-\{RA\}}(D).$$

3-Rough Topology:

The rough topology induced by an approximation space is defined through lower and upper approximations, where the lower approximation acts as an interior operator and the upper approximation acts as a closure operator. This topological interpretation enhances the structural understanding of uncertainty and supports rigorous classification.

Suppose U is a non-empty universe of objects and R is an equivalence relation on U, $X \subseteq U$. The family of equivalence classes $[x]_R$ form a base for a topology on U called rough topology; where, $\underline{R}(X) = int(X)$, $\overline{R}(X) = cl(X)$.

Definition 3.1[5] For $X \subseteq U$,we define the rough topology by $J_R = \{U, \emptyset, \overline{R}(X), \underline{R}(X), B_R(X)\}$

It is clear that:

1- U and $\emptyset \in J_R$.
2- since $\underline{R}(X) \subseteq \overline{R}(X), \underline{R}(X) \cup \overline{R}(X) = \overline{R}(X) \in J_R$

Also $\underline{R}(X) \cup B_R(X) = \overline{R}(X) \in J_R$ and $\overline{R}(X) \cup B_R(X) = \overline{R}(X) \in J_R$

Also $\underline{R}(X) \cap \overline{R}(X) = \underline{R}(X) \in J_R$: $\overline{R}(X) \cap B_R(X) = B_R(X) \in J_R$ and $\underline{R}(X) \cap B_R(X) = \emptyset \in J_R$.

Proposition 3.1[5]: The rough topology $J_R = \{U, \emptyset, \overline{R}(X), \underline{R}(X), B_R(X)\}$, where $X \subseteq U$ is a topology on U with respect to X.

Proof:

It is clear from definition (3.1), the following axioms are satisfies

1- U and $\emptyset \in J_R$.
2- Arbitrary union of any sub collction of $J_R \in J_R$.
3- Finite intersection of any sub collction of $J_R \in J_R$.

$J_R = (U, J_R, X)$ is called the rough topological space.

Example 3.1: Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and the relation R on U is $a \equiv b(mod3)$ for all $a, b \in U$ the equivalence classes are

$$U/R = \{\{0, 3, 6\}, \{1, 4, 7\}, \{2, 5, 8\}\}$$

Let $X = \{0, 2, 3, 5, 8\}, Y = \{1, 4, 7\}$. then:

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$\underline{R}(X) = \{2, 5, 8\}, \overline{R}(X) = \{0,2,3,5,6,8\}, B_R(X) = \{0,3,6\}$
$J_R(X) = \{U, \emptyset, \overline{R}(X), \underline{R}(X), B_R(X)\}$
$POS_R(X) = \{2, 5, 8\}, NEG_R(X) = \{1, 4, 7\}, \alpha_R(X) = 3\backslash 6 = 0.5$.
Also, $\underline{R}(Y) = \{1, 4, 7\}, \overline{R}(Y) = \{1, 4, 7\}, B(Y) = \{\emptyset\}$,
$POS_R(Y) = \{1, 4, 7\}$, $NEG_R(Y) = \{0, 2, 3, 5, 6, 8\}, \alpha_R(Y) = 3\backslash 3 = 1$.
$J_R(Y) = \{U, \emptyset, \overline{R}(Y), \underline{R}(Y), B_R(Y)\}$.

The relation R divides the set J_R into three classes. Through these classes, the rough approximations of the two sets X and Y are obtained.
The set X contains only one class that is fully included inside it, and it has different lower and upper approximations. Therefore, it is rough. On other hand, the set Y has a complete equivalence class that contains lower and upper approximations with the same value, and thus we conclude that the set is definable and does not contain a boundary region, and the negative region contains all the elements that are outside the class.
Remark 3.1: In rough set theory, we can find the topology induced by closure operator. This topology is constructed using upper approximations. The upper approximation acts as a Kuratowski closure operator, satisfying the conditions of a topological closure.
A set X is closed in this topology if its upper approximation of X is equal to X (i.e. $\overline{R}(X) = X$).
We find all subsets $X \subseteq U$ those definable sets. The collection of these definable sets forms a topology and denoted as $J_{(R)_c}$. In addition, $J_{(R)_c} = J_R$ as shown in the next theorem, which meaning the topology induced by the closure operator is identical to the topology generated by the equivalence classes.
Theorem 3.1: If the pair (U, R) is Pawlak approximation space, then $J_{(R)_c} = J_R$
(J_R is the original topology and $J_{(R)_c}$ is the topology induced by the closure operator).
Proof: Let $\in J_{(R)_c}$. Then A is a definable set, so $A_1, A_2, \ldots A_n \in (U/R)$ ،where $A = A_1 \cup A_2 \cup \ldots \cup A_n$ hence $A \in J_R$. That is $J_{(R)_c} \subseteq J_R$. Conversely, every $B \in J_R$ is union of some elements

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of (U/R) , which are definable. Since union of definable sets is again definable, B is definable this means $B \in J_{(R)_c}$ so $J_R \subseteq J_{(R)_c}$ as required.

Theorem3.2: Let (U, R) be Pawlak approximation space, X is subset of U, then $\overline{R}(X)$ is an empty set if and only if X does not contain any nonempty element of J_R .
Proof: The proof follows directly from definition of $\overline{R}(X)$.

4-Medical Information System for SLE:

4.1Decision Table:

In this section, we present a case study of ten patients suspected of having systemic lupus erythematosus (SLE). This disease is characterized by its complex nature, as its symptoms overlap with those of other diseases, such as anemia. Medically, these diseases share similar symptoms such as fatigue, exhaustion, and hair loss.

Because of the reduced red blood cell count, lupus is also likely to be accompanied by joint pain, chest pain, and hair loss. This similarity in symptoms makes it more difficult to distinguish between the two diseases, especially in their early stages.

The following symptoms were selected based on internationally accepted clinical criteria for SLE**:**

1- Joint pain and Swelling (Rheumatoid Arthritis) (RA).
2- Butterfly-shaped facial rash (BSFR).
3- Fatigue (FA).
4- Fever (FE).
5- Hair loss (HL).
6- Sensitivity to sunlight (STS).
7- Systemic Lupus Erythematosus (SLE).

The decision attribute is D: Diagnosis (SLE, Non SLE).

The condition attributes are ten symptoms listed above. The constructed decision table represents real-life patient cases U and reflects genuine diagnostic uncertainty.

In the next table, $U = P_1, \dots, P_{10}$ represent the patients, while the columns represent the symptoms, and the last column SLE is the decision attribute.

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Table (2): Medical information for some patients suspected of having systemic lupus erythematosus

Patient	RA	BSFR	FA	FE	HL	STS	SLE
P_1	Yes	Yes	No	Yes	Yes	Yes	Yes
P_2	Yes	Yes	Yes	Yes	Yes	Yes	Yes
P_3	Yes	Yes	No	Yes	Yes	No	No
P_4	Yes	No	No	Yes	No	No	No
P_5	Yes	Yes	No	Yes	No	No	No
P_6	Yes	Yes	Yes	No	No	Yes	Yes
P_7	No	Yes	No	No	No	Yes	Yes
P_8	No	Yes	No	No	No	Yes	No
P_9	No	No	No	No	Yes	No	Yes
P_{10}	No	Yes	Yes	No	No	Yes	Yes

4.2 Medical Interpretation:

One of the advantages of the table is that it allows for a thorough examination of each patient's symptoms and the formulation of a diagnosis based on them, for example.

For patient P_1 :
RA: Yes, BSFR: Yes, FA: No, FE: Yes, HL: Yes, STS: Yes , and SLE: Yes.
Diagnosis: according to the information shown, he is suffering from systemic lupus erythematosus (SLE).

For patient P_4 :
RA: Yes, BSFR: No, FA: No, FE: Yes, HL: No, STS: No.

Diagnosis: according to the information shown, he is not infected.

The table above clearly shows some patients with similar symptoms but different diagnoses. For example, patients P7 and P8, despite identical symptoms, received completely different diagnoses, even though all medical conditions were the same, and the same treating physician may have even made the diagnosis. The medical explanation might be that the patient has another very similar disease, such as anemia or mixed connective tissue disease. This necessitates the use of more precise and rigorous methods to analyze such discrepancies. In this research, we will employ rough topology technique in section 5 to resolve this ambiguity and arrive at sound and accurate medical interpretations, results, and decisions.

Remark 4.1: If we consider each symptom individually, for example (RA) or (FE) we can classify patients into two specific symptoms sets:

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Those who have RA: $X = \{P_1, P_2, P_3, P_4, P_5, P_6\}$.
Those who do not have RA: $Y = \{P_7, P_8, P_9, P_{10}\}$.
Also, those who have FE: $X = \{P_1, P_2, P_3, P_4, P_5\}$.
Those who do not have FE: $Y = \{P_6, P_7, P_8, P_9, P_{10}\}$.

Based on the preceding discussion and Table (2), instead of analyzing each column individually in the table, we can simplify the process using the rough set technique, focusing on the reduct and core, which represent the essential and necessary attributes for decision-making. This allows us to reduce the number of columns without losing the ability to distinguish between patients, while maintaining analysis accuracy, and these are the concepts that we will study later.

Measures Applied to the Medical Table: Rough Topology5-

In this section, we apply concepts of rough topology to the medical data presented in table (2), which shows the possible symptoms of ten patients suspected of having systemic lupus erythematosus (SLE). SLE was chosen due to its complex nature. This disease is characterized by the overlap of its symptoms with other diseases such as anemia, where both share symptoms like fatigue, exhaustion, and hair loss. Lupus disease may also appear with joint pain, chest pain, and hair shedding due to red blood cell deficiency and this similarity in symptoms leads to difficulties in distinguishing between the two diseases in the early stages. Therefore, we use rough topology techniques to obtain better decisions in diagnosing the disease.

$U = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}\}$ be the set of patients.
$C = \{RA, BSFR, FA, FE, HL, STS\}$ be the set of condition attributes.
$D = \{SLE, Non - SLE\}$ be the decision attribute.
$J_R(X) = \{U, \emptyset, \overline{R}(X), \underline{R}(X), B_R(X)\}$.

Based on the medical table, the indiscernibility relation generated by C partitions the patients into equivalence classes. Patients P_7 and P_8 share identical symptom profiles, while the remaining patients form singleton equivalence classes.

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5.1Equivalence Classes

Each symptom induces an equivalence relation on the patient set. The intersection of all condition attribute relations generates the indiscernibility relation

$$U/RA = \{P_1, P_2, P_3, P_4, P_5, P_6\}, \{P_7, P_8, P_9, P_{10}\}$$
$$U/BSFR = \{\{P_1, P_2, P_3, P_5, P_6, P_7, P_8, P_{10}\}, \{P_4, P_9\}\}$$
$$U/FA = \{\{P_1, P_3, P_4, P_5, P_7, P_8, P_9\}, \{P_2, P_6, P_{10}\}\}$$
$$U/FE = \{\{P_1, P_2, P_3, P_4, P_5\}, \{P_6, P_7, P_8, P_9, P_{10}\}\}$$
$$U/HL = \{\{P_4, P_5, P_6, P_7, P_8, P_{10}\}, \{P_1, , P_2, P_3, P_9\}\}$$
$$U/STS = \{\{P_1, P_2, P_6, P_7, P_8, P_{10}\}, \{P_3, P_4, P_5, P_9\}$$

Taking intersection of six equivalence relations, we get:

$$U\backslash C = \left\{\{P_7, P_8\}, \{P_1\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_5\}, \{P_6\}, \{P_9\}, \{P_{10}\}\right\}.$$

5.2. Tools of rough topology:

For the set U of patients and the attribute D , we have

$$U/D = \{\{P_1, P_2, P_6, P_7, P_9, P_{10}\}, \{P_3, P_4, P_5, P_8\}$$

The set of individuals diagnosed with the disease $X = \{P_1, P_2, P_6, P_7, P_9, P_{10}\}1$.

The set of individuals without the disease $Y = \{P_3, P_4, P_5, P_8\}$ 2.

$\overline{R}(X) = \{P_1, P_2, P_6, P_7, P_8, P_9, P_{10}\}$, $\underline{R}(X) = \{P_1, P_2, P_6, P_9, P_{10}\}$,

$B_R(X) = \{P_7, P_8\}$, $POS_R(X) = \{P_1, P_2, P_6, P_9, P_{10}\}$, $NEG_R(X) = \{P_3, P_4, P_5\}$

$J_R(X) = \left\{U, \emptyset, \overline{R}(X), \underline{R}(X), B_R(X)\right\}$.

Also, $\overline{R}(Y) = \{P_3, P_4, P_5, P_7, P_8\}$, $\underline{R}(Y) = \{P_3, P_4, P_5\}$, $B_R(Y) = \{P_7, P_8\}$,

$POS_R(Y) = \{P_3, P_4, P_5\}$, $NEG_R(Y) = \{P_1, P_2, P_6, P_9, P_{10}\}$,

$J_R(Y) = \left\{U, \emptyset, \overline{R}(Y), \underline{R}(Y), B_R(Y)\right\}$.

5.3. Accuracy of Approximation:

Using the condition attributes, (RA, FE, FA, BSFR, STS, and HL), the indiscernibility relation generated nine equivalence classes.

For the positive class(SLE = Yes), number of the lower approximation is 5 and number of the upper approximation is 7, so:

The accuracy of X : $\alpha_R(X) = \frac{5}{7} = 0.714$.

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For the negative class Y (SLE = No), number of the lower approximation is 3 and number of the upper approximation is 5, so:
The corresponding accuracy of $Y : \alpha_R(Y) = \frac{3}{5} = 0.6$.
So, the calculated approximation accuracy values is
(0.714 for SLE = Yes and 0.6 for SLE = No) demonstrate that the physician's diagnostic decisions are largely supported by the selected clinical attributes, while simultaneously revealing the presence of a measurable degree of uncertainty. The existence of boundary region objects does not indicate an error in the medical diagnosis rather, it reflects the inherent overlap of clinical and laboratory features among certain patients.

From rough topological perspective, the lower approximations represent diagnostically stable cases where the physician's decision is strongly supported by the available data, whereas the boundary region identifies cases that require closer clinical observation or additional medical investigation. Therefore, rough topology proves to be an effective mathematical tool for evaluating diagnostic confidence, quantifying uncertainty, and highlighting critical cases without altering the physician's original judgment.

5.4. Positive Region and Dependency Degree

I-Positive Region:

The cases that can be determined with certainty based on the available attributes; the more this region increases, the more it indicates the clarity and strength of the attributes in decision-making. We calculate the classification importance from the decision column, then we calculate the lower approximation for each decision set from the attribute column, and then we take the union of all lowers to find the positive region.

Since C − positive region of D: $POS_C(D) = \bigcup_{Y \in IND(D)} \underline{C}(Y)$

$$U \backslash C = \{\{P_7, P_8\}, \{P_1\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_5\}, \{P_6\}, \{P_9\}, \{P_{10}\}\}$$

$U/D = \{\{P_1, P_2, P_6, P_7, P_9, P_{10}\}, \{P_3, P_4, P_5, P_8\}\}$
$X = \{P_1, P_2, P_6, P_7, P_9, P_{10}\}, Y = \{P_3, P_4, P_5, P_8\}$
$\underline{C}(X) = \{M \in IND(C) : M \subseteq X, X \in IND(D)\}$
$= \{P_1, P_2, P_6, P_9, P_{10}\}$.
$\underline{C}(Y) = \{M \in IND(C) : M \subseteq Y, Y \in IND(D)\} = \{P_3, P_4, P_5\}$.
Then:

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$$POS_C(D) = \{P_1, P_2, P_6, P_9, P_{10}\} \cup \{P_3, P_4, P_5\}$$
$$= \{P_1, P_2, P_3, P_4, P_5, P_6, P_9, P_{10}\}$$

II- Dependency Degree:

As we explained previously, dependency degree is defined as the number of elements in the positive region divided by total number of elements so.

$$k = \gamma_C(D) = \frac{cardPOS_C(D)}{card\ U} = \frac{8}{10} = 0.8.$$

Since $0 < k = 0.8 < 1$

Hence, the decision attribute roughly (partially) depends on attributes C.

III- Medical Interpretation:

In this dataset, each patient is described by clinical attributes: (RA, FE, FA, BSFR, STS, HL) and classified according to SLE diagnosis. Eight patients can be classified with complete certainty based on these attributes, forming the positive region. Two patients share identical clinical features but have conflicting decisions, forming the boundary region, reflecting real diagnostic ambiguity. The dependency degree of the decision on the condition attributes is $\gamma(B, D) = 0.8$, indicating that 80% of the decision outcomes are deterministically determined by the attributes, while 20% remain uncertain, consistent with the clinical complexity of SLE.

5.5. Attributes Importance:

Now, we are calculating attributes importance to help us identify unnecessary features that can be removed without affecting the accuracy of the diagnosis. In addition, it improves the accuracy of the decision model.

$SIG(RA, C, D) = \gamma_C(D) - \gamma_{C-\{RA\}}(D)$

First, we calculate $POS_{C-\{RA\}}(D)$ we have

$$POS_{C-\{RA\}}(D) = \bigcup_{X \in IND(D)} \underline{C - \{RA\}}\,(X)$$

$U(C - \{RA\})$

$= \{\{P_6, P_{10}\}, \{P_7, P_8\}, \{P_1\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_5\}, \{P_9\}\}$

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1- $X = \{P_1, P_2, P_6, P_7, P_9, P_{10}\}$

$\underline{C - \{RA\}}(X) = \{M \in IND(C - \{RA\}) : M \subseteq X, M \in IND(D)\}$
$= \{P_1, P_2, P_6, P_9, P_{10}\}$.

2- $Y = \{P_3, P_4, P_5, P_8\}$

$\underline{C - \{RA\}}(Y) = \{M \in IND(C - \{RA\}) : X \subseteq Y, Y \in IND(D)\}$
$= \{P_3, P_4, P_5\}$

SO, $POS_{C-\{RA\}}(D) = \cup_{Y \in IND(D)} \underline{C - \{RA\}}\, Y = \{P_1, P_2, P_6, P_9, P_{10}\} \cup \{P_3, P_4, P_5\} = \{P_1, P_2, P_3, P_4, P_5, P_6, P_9, P_{10}\}$.

Second: we calculate $k = \gamma_{C-\{RA\}}(D)$

$$= \frac{cardPOS_C(D)}{card\ U} = \frac{8}{10} = 0.8.$$

Therefore $SIG(RA, C, D) = \gamma_C(D) - \gamma_{C-\{RA\}}(D) = 0.8 - 0.8 = 0.$

Since $k = 0$ then D totally independent from $C - \{RA\}$.
Similarly, we can calculate the importance of each attribute.

5.6. Applying Reducts and Core Attributes:

To identify reducts, each symptom attribute was removed individually from the condition attribute set C, and the dependency degree γ was recalculated. We will do this in detail in the following steps.

Step1: If we remove the attribute (RA) from the condition attribute set, we obtain the family of equivalence classes corresponding to the resulting attribute set, which is

$$U \backslash (C - \{RA\}) = \{\{P_6, P_{10}\}, \{P_7, P_8\}, \{P_1\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_5\}, \{P_9\}\} \neq U/C$$

The attribute RA is indispensable.

When the attribute RA was removed, a change in the classification decision was observed. Some patients became indiscernible while having different SLE outcomes. Therefore, the dependency degree decreased. This indicates that RA is a core attribute.

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Step2: If we remove the attribute (BSFR) from the condition attribute set, we obtain the family of equivalence classes corresponding to the resulting attribute set, which is

$$U\backslash(C-\{BSFR\}) = \{\{P_4, P_5\}, \{P_7, P_8\}, \{P_1\}, \{P_2\}, \{P_3\}, \{P_6\}, \{P_9\}, \{P_{10}\}\} \neq U/C$$

The attributeBSFR is indispensable.

When the BSFR attribute was removed, a change in the classification decision was observed; the reliability score decreased. This indicates that BSFR is a fundamental attribute, just like its predecessor.

Step3: If we remove the attribute (FA) from the condition attribute set, we obtain the family of equivalence classes corresponding to the resulting attribute set, which is

$$U\backslash(C-\{FA\}) = \{\{P_1, P_2\}, \{P_7, P_8\}, \{P_3\}, \{P_4\}, \{P_5\}, \{P_6\}, \{P_9\}, \{P_{10}\}\} \neq U/C$$

The attribute FA is indispensable

Similarly, when the FA attribute was removed, a change in the classification decision was observed. Consequently, the reliability score decreased. This indicates that FA is a fundamental attribute.

Step 4: If we remove the attribute (FE) from the condition attribute set, we obtain the family of equivalence classes corresponding to the resulting attribute set, which is

$$U\backslash(C-\{FE\}) = \{\{P_7, P_8\}, \{P_1\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_5\}, \{P_6\}, \{P_9\}, \{P_{10}\}\} = U/C$$

The attribute FE is dispensable.

Step5: If we remove the attribute HL from the condition attribute set, we obtain the family of equivalence classes corresponding to the resulting attribute set, which is

$$U\backslash(R-\{HL\}) = \{\{P_7, P_8\}, \{P_1\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_5\}, \{P_6\}, \{P_9\}, \{P_{10}\}\} = U/C$$

The attribute HL is dispensable.

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When attributes FE, HL was removed, it did not affect the dependency degree. The classification results remained unchanged. Therefore, these attributes are not core attributes.

Step6: If we remove the attribute (STS) from the condition attribute set, we obtain the family of equivalence classes corresponding to the resulting attribute set, which is

$$U\backslash(R-\{STS\}) = \{\{P_1, P_3\}, \{P_7, P_8\}, \{P_2\}, \{P_4\}, \{P_5\}, \{P_6\}, \{P_9\}, \{P_{10}\}\} \neq U/C$$

The attribute STS is indispensable.

When the attribute STS was removed, a change in the classification decision was observed. Some patients became indiscernible while having different SLE outcomes. Therefore, the dependency degree decreased. This indicates that STS is a core attribute.

Now we will calculate the reduct and the core.

Since, the attributes RA, BSFR, FA, STS is indispensable in C.

That means that the classification defined by the set of six equivalence attributes RA, BSFR, FA, FE, STS, HL is the same as the classification defined by attributes RA, BSFR, FA, STS and FE or RA, BSFR, FA, STS and HL. To find reducts of the C we must check whether pairs of attributes(RA, BSFR, FA, STS) , (FE) and (RA, BSFR, FA, STS) ,(HL) are independent or not.

Because $IND(\{(RA, BSFR, FA, STS), (FE)\}) \neq IND(FE)$ and $IND(\{(RA, BSFR, FA, STS), (HL)\}) \neq IND(HL)$, hence the attributes (RA, BSFR, FA, STS) and (FE) are independent, and consequently {(RA, BSFR, FA, STS), (FE)) is a reduct ofC. Proceeding in the same way we find that (RA, BSFR, FA, STS) ,(HL) is also a reduct of C.

Thus, there are two reducts ofC , namely{(RA, BSFR, FA, STS) , (FE)} and {(RA, BSFR, FA, STS) ,(HL)}

The core of C ={(RA, BSFR, FA, STS) , (FE)} ∩ {(RA, BSFR, FA, STS) ,(HL)}

= (RA, BSFR, FA, STS) .

5.7. General Medical Interpretation:

1- Through all the previous procedures and steps, it has become possible to conclude that the (RA) feature (rheumatoid arthritis) is

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considered a decisive clinical indicator within the mentioned group of symptoms, as autoimmune interference and patterns of arthritis are often associated with systemic immune disorders, as was clearly demonstrated in the diagnosis of this disease. Similarly, all the symptoms (FA, STS, and BSFR) appeared as essential features indispensable for maintaining complete diagnostic accuracy, making them form the core of the decision system. Their presence contributes significantly to distinguishing affected patients from unaffected ones. It was also confirmed that the removal of any of them plays a major role in reducing diagnostic accuracy. Meanwhile, it was observed that the remaining symptoms represented by (FE, HL) contributes supportive clinical information but is not individually decisive in determining the diagnosis, as its removal did not affect diagnostic accuracy, making it outside the core of the system.

2- As shown by classifying the patients in the attached table, into two groups, those affected (X) and not affected (Y). Based on the lower and upper approximations of the affected group (X), the results show that the approximation accuracy is 0.714, indicating that about 71.4% of patients affected by the disease can be diagnosed accurately, while the remaining cases fall within the boundary range due to unclear symptoms. Similarly, for the patients not affected by the disease (Y), the approximation accuracy reached 0.6. The reliability index is 0.8, indicating that about 80% of all patients in this group can be accurately classified using the conditional attributes, thereby supporting medical decision-making and reducing uncertainty in diagnosis.

6. Conclusion

Rough topology techniques are characterized by high quality and great accuracy, especially when applied in practical matters, particularly in medical data. This became evident in this study when they were applied to systemic lupus erythematosus, where they proved effective in analyzing cases of uncertainty and overlap. The basic diagnostic features of the disease were also identified through the application of the tools. The results showed that only some symptoms are sufficient to diagnose the disease, as it was found that joint pain and swelling, the butterfly-shaped rash on the face, sensitivity to light, were among the most influential features in the

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decision-making process. Meanwhile, the remaining symptoms are not of significant importance for obtaining an accurate classification through the elimination process. Additionally, the lower approximation identified confirmed cases displaying clear manifestations of the disease, whereas the upper approximation included borderline cases with partial or ambiguous symptoms that require additional tests or regular follow-up.

These modern tools have enabled better management of ambiguity in real clinical scenarios. Overall, the results confirm that topological approximation techniques provide high diagnostic accuracy for better medical decision-making.

References:

[1] Pawlak, Z. (1982). Rough sets. Information Sciences, 11(5), 341–356.

[2] Orlowska, E. (1998). Incomplete Information: Rough Set Analysis. Heidelberg: Physica-Verlag.

[3] Pawlak, Z. (1991). Rough Sets: Theoretical Aspects of Reasoning about Data. Dordrecht :Springer.

[4] Yao, Y. Y. (1998). Relational interpretations of neighborhood operators. Information Sciences, 111(1–4), 239–259.

[5] Pawlak, Z., & Skowron, A. (2007). Rough set: Some extensions. Information Sciences, 177, 28–40.

[6] Rahman, A., & Isenberg, D. A. (2008). Systemic lupus erythematosus. New England Journal of Medicine, 358(9), 929–939.

[7] Durcan, L., O'Dwyer, T., & Petri, M.(2019) .Management strategies and future directions for systemic lupus erythematosus.

[8] MedlinePlus Genetics. (2024). Systemic lupus erythematosus. https://medlineplus.gov/download/genetics/condition/systemic-lupus-erythematosus.

[9] Tsokos, G. C. (2025). Systemic lupus erythematosus update. Signal Transduction and Targeted Therapy, 10, 102.

[10] Tian, J., & Zhang, D. (2023). Global epidemiology of SLE. Lupus Science & Medicine, 10(1), e000834.

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http://www.doi.org/10.62341/istj-vol38-2-AN76

[11] Pons-Estel, G. J., Alarcón, G. S., et al. (2024). Multiple faces of SLE. Mediterranean Journal of Rheumatology, 35, 319–327.
[12] Hochberg, M. C. (1997). Updating ACR criteria for SLE. Arthritis & Rheumatism, 40(9), 1725.
[13] Petri, M., Orbai, A. M., et al. (2012). SLICC classification criteria for SLE. Arthritis & Rheumatism, 64(8), 2677–2686.

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